Experiment 1: Spectral Analysis

Objective

In this experiment you will learn how to use a spectrum analyzer by working with periodic (sine, square and triangular) waveforms. You will observe how waveform parameters affect the spectra of corresponding waveforms and verify time/frequency scaling properties of the Fourier transform using a rectangular pulse. You will use spectral analysis to identify non-linear systems.

Prelab Assignment

1. Determine the exponential Fourier series coefficients of the following periodic waveforms.

(a) $x_s(t) = A\cos 2\pi f_0 t$.

(b) $x_w(t)$ is the square wave shown in Figure (1a) with amplitude A, frequency f_0 and pulse duration τ, where $T_0 = 1/f_0$ is the period.

(c) $x_\Delta(t)$ is the triangular wave shown in Figure (1b) with amplitude A and frequency f_0, where $T_0 = 1/f_0$ is the period.

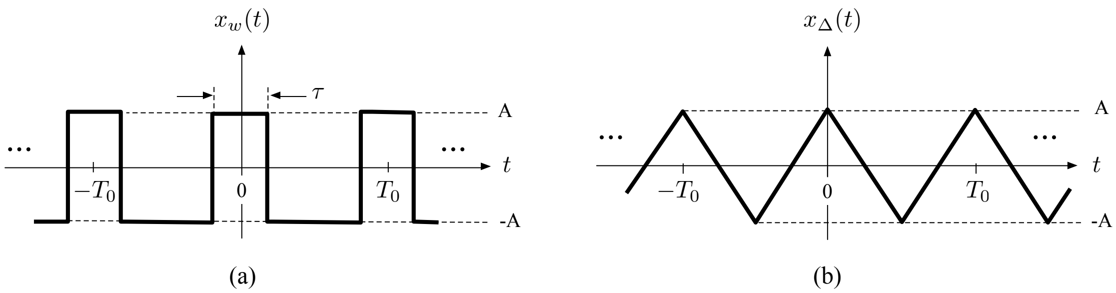

Figure 1: Periodic waveforms: **(a)** Square wave $x_w(t)$ and **(b)** Triangular wave $x_\Delta(t)$.

(d) From the results you obtained in parts **(a)–(c)**, compute the numeric values of the spectral components of $x_s(t)$, $x_w(t)$ and $x_\Delta(t)$ for $A = 1$ V, $f_0 = 2$ kHz and $\tau = T_0/5$. Plot the corresponding *one-sided rms-spectra*[1] using spectral components upto and including the 6*th* harmonic term.

2. Let $x(t) = A\,\textbf{rect}(t/\tau)$ be a rectangular pulse[2] centered at the origin with amplitude A and pulse width τ. Determine $X(f) = \mathcal{F}[x(t)]$ and sketch the resulting magnitude spectrum $|X(f)|$. Label the spectral nulls, i.e., frequency values where $|X(f)| = 0$, in terms of τ.

[1] For a definition of *one-sided rms-spectrum* refer to *Course Notes Part 1*, pp. 19–20.

[2] A rectangular pulse with pulse duration τ is defined as

$$\textbf{rect}(t/\tau) = \begin{cases} 1, & |t| \le \tau/2; \\ 0, & \text{otherwise.} \end{cases}$$

3. Consider the systems $\mathcal{S}_1$ and $\mathcal{S}_2$ with input-output relations:

$$\begin{aligned} \mathcal{S}_1 &: y_1(t) = \alpha x(t); \\ \mathcal{S}_2 &: y_2(t) = \alpha x^2(t) + \beta x(t); \end{aligned} \tag{1}$$

where $\alpha, \beta \in \mathbb{R}$. $x(t)$ is the input, $y_1(t)$ and $y_2(t)$ are the outputs of $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively. Let $x_1(t) = A_1 \cos 2\pi f_1 t$ and $x_2(t) = A_2 \cos 2\pi f_2 t$ with $0 < f_1 < f_2$.

(a) Determine the linearity of the systems $\mathcal{S}_1$ and $\mathcal{S}_2$. Justify your answers.

(b) Determine $y_1(t)$ and $y_2(t)$ when $x(t) = x_1(t)$.

(c) Determine $y_1(t)$ and $y_2(t)$ when $x(t) = x_2(t)$.

(d) Determine $y_1(t)$ and $y_2(t)$ when $x(t) = x_1(t) + x_2(t)$.

Equipment

In this experiment you will use the following equipment and software:

- Agilent DSO-X 2002A digital storage oscilloscope with waveform generation and spectrum analyzer options.
- Computer with Linux operating system.
- Matlab/Simulink 2014b.

Procedure

In Part-A and Part-B of this experiment you will use the internal function generator of the Agilent DSO-X 2002A digital storage oscilloscope. See Figure (2) for reference.

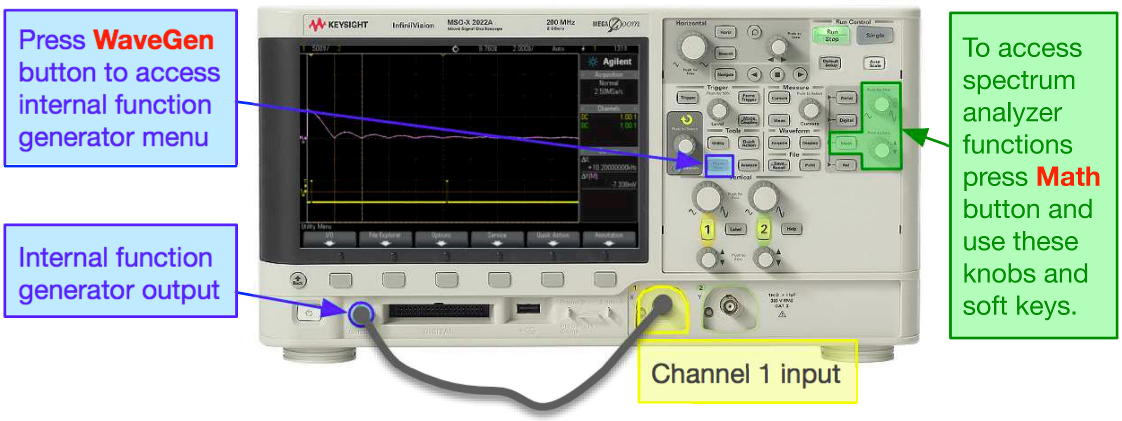

Figure 2: Connecting the internal function generator of DSO-X 2002A digital storage oscilloscope.

Familiarize yourself with the controls of the oscilloscope. It is important that you learn the key features of the oscilloscope, set-up procedures, spectrum analyzer functions and how to capture images of displayed waveforms. Your instructor will provide you with a brief introduction to get you started. The complete *User Manual* is available on BlackBoard.

Important: The built-in spectrum analyzer operates in AC coupled mode and therefore does not display spectral components corresponding to the DC components of the signals being analyzed. For example, if the periodic input signal $x(t)$ with period $T_0 = 1/f_0$ and non-zero DC value has the Fourier series expansion:

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2\pi n f_0 t},$$

and the corresponding Fourier transform:

$$X(f) = \mathcal{F}[x(t)] = \sum_{n=-\infty}^{\infty} D_n \delta(f - n f_0).$$

In this example $D_0 \neq 0$ since $x(t)$ has a non-zero DC value. However, the first spectral component of the one-sided RMS spectrum displayed on the screen is **located at** f_0 even though $D_0 \neq 0$.

Part-A/B Setup

Function Generator: Select internal function generator of DSO-X 2002A. Press **[WaveGen]**. Settings specific to each waveform are listed below.

Spectrum Analyzer: Press **[Math]** $>$ **[Operator:** FFT**]**. Use the following control settings: **[Span:** 50 kHz**]**, **[Center:** 25 kHz**]**, **[Window:** Rectangle**]**, **[Vertical Units:** V rms**]**. The last two control settings are accessible by pressing the **[More FFT]** softkey.

A. Line spectra of periodic signals

> **Part-A Preset:** Oscilloscope settings used in Part-A are stored in the file **e1setupA.scp**. To use the preset values: Press **[Save/Recall]** > **[Recall]** > **[Load from:** e1setupA**]**.

1. Adjust the function generator to output a **sine wave** with amplitude $A = 1$ V (corresponding to *peak-to-peak* amplitude of 2 V_{pp}) and frequency $f_0 = 2$ kHz. Display the signal and its magnitude spectrum on the oscilloscope. Record the amplitudes and frequencies of the spectral components. Change the amplitude and frequency of the input signal and observe the corresponding changes in spectral components.

2. Repeat Step A.1 using a **square wave** with amplitude $A = 1$ V (corresponding to *peak-to-peak* amplitude of 2 V_{pp}), frequency $f_0 = 2$ kHz and 20% duty cycle[3] as shown in Figure (1a).

3. Change the **duty cycle** of the square wave described in Step A.1 to 50% and then to 80%. Observe the effect of duty cycle on the spectral components. Record/plot the resulting magnitude spectra.

4. Repeat Step A.1 using a **ramp/triangular wave** with amplitude $A = 1$ V (corresponding to *peak-to-peak* amplitude of 2 V_{pp}) and frequency $f_0 = 2$ kHz as shown in Figure (1b).

Problem A.1 Tabulate the amplitudes and frequencies of the spectral components you measured in Steps A.1, A-2 and A.4 and calculated in Prelab Assignment Question 1. Compare the two sets of results and comment on differences observed.

Problem A.2 Let $x_w(t)$ be the square wave generated by the function generator and let $|X_w(f)|$ be its magnitude spectrum. Plot $|X_w(f)|$ for 20% and 50% duty cycles as determined in Step A.2–3. Discuss how the spectral nulls of the envelope of $|X_w(f)|$ and the separation between the spectral components change as a function of duty cycle.

Problem A.3 Let $x_p(t)$ be rectangular pulse[4] extracted from $x_w(t) + A$. Let $|X_p(f)|$ be its magnitude spectrum. Compare $|X_p(f)|$ with the envelope of $|X_w(f)|$ (use the results of Prelab Assignment Question 2).

Problem A.4 Comment on the differences (if any) of the magnitude spectra of the square waves with 20% and 80% duty cycles. Explain your answer.

Problem A.5 Compare the frequency content of the sine, square (with 50% duty cycle) and triangular waves you investigated in Steps A.1, A.3 and A.4. Comment on differences observed.

[3] *Duty cycle* is defined as the percentage of one period in which a signal is positive (active), i.e., τ/T_0.

[4] The rectangular pulse $x_p(t)$ is defined as:

$$x_p(t) = \begin{cases} x_w(t) + A, & |t| \le T_0/2; \\ 0, & \text{otherwise.} \end{cases}$$

B. Spectrum of a rectangular pulse

> **Part-B Preset:** Oscilloscope settings used in Part-B are stored in the file **e1setupB.scp**. To use the preset values: Press **[Save/Recall]** > **[Recall]** > **[Load from:** e1setupB**]**.

1. Adjust the function generator to output a **rectangular pulse**[5] with amplitude $A = 2$ V and pulse duration $\tau = 100\ \mu$s. If necessary, change the time-base setting of the oscilloscope such that only one pulse is visible on the screen. Display and record the magnitude spectrum of the pulse.

   Note: If you have not used the preset settings in the setup file **e1setupB.scp**, you can generate the pulse using the following settings for the waveform generator: **[Frequency:** 100 Hz**]**, **[Amplitude:** 2 V_{pp}**]**, **[Offset:** 1 V**]**, **[Width:** 100 μs**]**.

2. Change the pulse duration τ and observe the corresponding changes in the magnitude spectrum (make sure that only one pulse is visible on the screen at all times). In particular, set the pulse duration to 50 μs and then to 200 μs; record the resulting magnitude spectra.

Problem B.1 Compare the plots you recorded in Step B.1 and B.2 with the results of Prelab Assignment Question 2.

Problem B.1 Determine how the he spectral nulls of the magnitude spectra you generated in Steps B.1 and B.2 are related to the pulse width τ.

C. Testing system linearity using spectral analysis

> **Part-C Setup**
>
> **Function Generator:** Matlab/Simulink running the appropriate Simulink model.
>
> **Spectrum Analyzer:** Press **[Math]** > **[Operator:** FFT**]**. Use the following control settings: **[Span:** 50 kHz**]**, **[Center:** 25 kHz**]**, **[Window:** Rectangle**]**, **[Vertical Units:** V rms**]**. The last two control settings are accessible by pressing the **[More FFT]** softkey.
>
> **Preset:** Oscilloscope settings used in Part-C are stored in the file **e1setupC.scp**. To use the preset values: Press **[Save/Recall]** > **[Recall]** > **[Load from:** e1setupC**]**.

In Part-C of this experiment you will use Matlab/Simulink as a signal generator. Simulink models will generate signals that are copied to the sound-out port of the computer. By connecting the sound-out port to the oscilloscope you will be able to monitor Simulink generated signals on the oscilloscope. See Figure (3) for reference.

[5] The function generator outputs a periodic waveform. However, the periodic output can mimic a single rectangular pulse if the output is a square wave with a slow repetition rate (frequency).

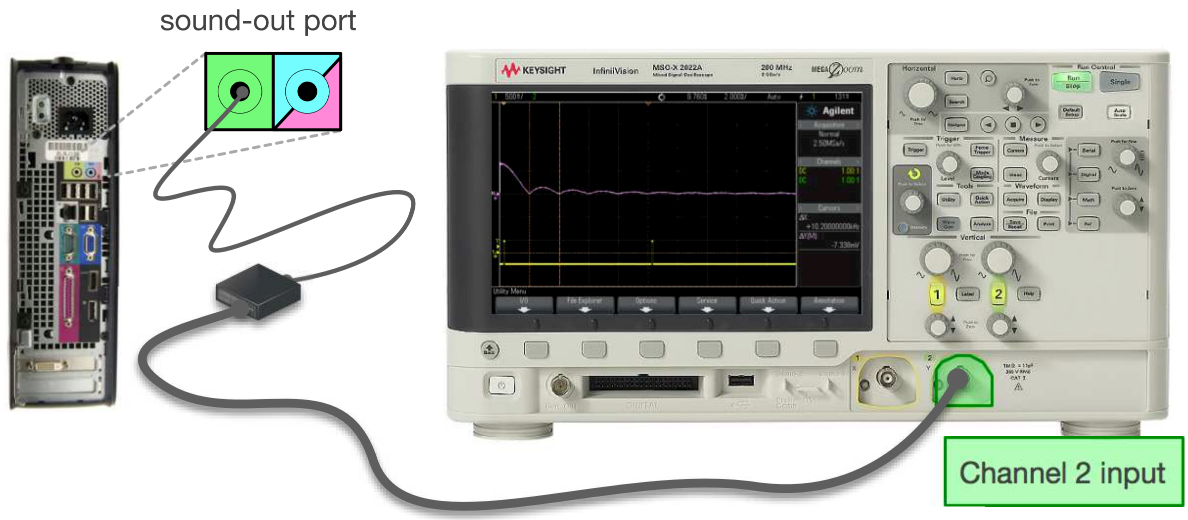

Figure 3: Connecting the sound-out port of the computer to DSO-X 2002A digital storage oscilloscope.

In this part of the experiment you will use the Simulink models **Exp1_C1.slx** and **Exp1_C2.slx**. Download the Simulink models from [BlackBoard] > [Laboratory] > [Experiment 1] to an appropriate location (e.g. "/home/student/name/ele635/exp1/") in your home directory.

1. Open and run the Simulink model **Exp1_C1.slx**. The function generator **Sine Wave1** in the Simulink model outputs a sine wave with frequency 1 kHz. Using the switch toggle between the outputs of the linear and non-linear systems. Observe and record the magnitude spectra of the outputs of both systems. **Note:** For Part-C of this experiment our focus is the frequencies of the spectral components rather than their magnitudes.

2. Open and run the Simulink model **Exp1_C2.slx**. In this step of the experiment you will monitor the outputs of *System A* and *System B*; one of the systems is *linear* and whereas the other one is *non-linear*. Function generators **Sine Wave1** and **Sine Wave2** output sine waves with equal amplitudes but different frequencies. Using the switch toggle between the outputs of the two systems. Observe and record the magnitude spectra of the outputs of both systems.

Problem C.1 Compare the plots you recorded in Step C.1. Comment on the effects of nonlinearity on the system output when the system input is a sine wave.

Problem C.2 From the plots you recorded in Step C.2 determine the frequencies of the two sine wave generators. Identify which of *System A* and *System B* is the *non-linear* system; explain your reasoning. In view of the the results of Prelab Assignment Question 3, what can you say about the nature of nonlinearity for the system that you identified as being non-linear.